

MODELING FREQUENCY SELECTIVE SURFACES WITH MULTIPLE METALLIZATION AND CONTINUOUSLY LAYERED DIELECTRIC SUBSTRATES

B.J. Morsink⁽¹⁾, S.W. Rienstra⁽²⁾ and A.G. Tijhuis⁽³⁾

⁽¹⁾*Thales Naval Nederland, Business Unit Radar and Sensors
P.O. Box 42, 7550 GD Hengelo (Ov.), the Netherlands
Email: bart.morsink@nl.thalesgroup.com*

⁽²⁾*Faculty of Mathematics and Computing Science, Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven, the Netherlands
Email: s.w.rienstra@tue.nl*

⁽³⁾*Faculty of Electrical Engineering, Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven, the Netherlands
Email: a.g.tijhuis@tue.nl*

INTRODUCTION

In a collaboration between Thales Naval Nederland and the Eindhoven University of Technology, a numerical approach is being developed for the analysis and design of frequency selective surfaces. By choosing a modular approach, we hope to obtain sufficient flexibility in the choice of the configuration. Multiple metallization layers as well as piecewise homogeneous and/or continuously stratified plane substrates can be handled. At the time of submission of this manuscript, the first numerical results were being generated and compared with experimental results. Therefore, we restrict ourselves to the general approach. Details of the implementation and representative results will be discussed at the conference. To keep the discussion tractable, we first consider a single metal layer on top of a layered dielectric substrate, and defer the generalization to multiple metal layers until the end of the paper.

FORMULATION OF THE PROBLEM

We consider a Bravais lattice of metal patches in the plane $z = 0$. In the half-space $z < 0$, we have free space, the layer $0 < z < d$ is a continuously layered dielectric slab, and in the half-space $z > d$, we have a homogeneous dielectric. The entire configuration is symmetric with respect to two basis vectors \mathbf{a}_1 and \mathbf{a}_2 , i.e., translation over either of these basis vectors moves the observation point between identical physical situations. The configuration is illuminated from above by an incident plane wave with wave vector \mathbf{k}^i . An impression of the configuration is given in Figure 1.

In our modeling approach, we use the linearity of the problem. We define $\mathbf{E}^i(\mathbf{r})$ as the electric field that would be present in absence of the metallization. The total electric field is then the superposition of $\mathbf{E}^i(\mathbf{r})$ and the electric field generated by the surface current $\mathbf{J}_S(\mathbf{r})$ that is induced on the metal patches. By requiring that the tangential component of both fields vanishes on the patches, we then arrive at an integral equation for $\mathbf{J}_S(\mathbf{r})$.

SPATIAL FOURIER TRANSFORMATION

We first determine the spectral representation of the field caused by an external current density

$$\mathbf{J}(\mathbf{r}) = \mathbf{J}_T(\mathbf{r}_T) \delta(z), \quad (1)$$

where the subscript T denotes a transverse component. The current is located in a dielectric medium with permittivity $\varepsilon(z)$, conductivity $\sigma(z)$ and permeability $\mu(z)$. To exploit the fact that the constitutive parameters in (2) depend only on the z -coordinate, we solve these equations in the spectral domain. To this end, we introduce the spatial Fourier transformation

$$\hat{\mathbf{E}}(\mathbf{k}_T, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r}) \exp(-i\mathbf{k}_T \cdot \mathbf{r}_T) dx dy. \quad (2)$$

This transformation reduces the gradient operator in (2) to $i\mathbf{k}_T + \mathbf{u}_z \partial_z$. We decompose the electromagnetic fields into their transverse and longitudinal parts, and we break up Maxwell's equations in the same manner. Next, the transverse

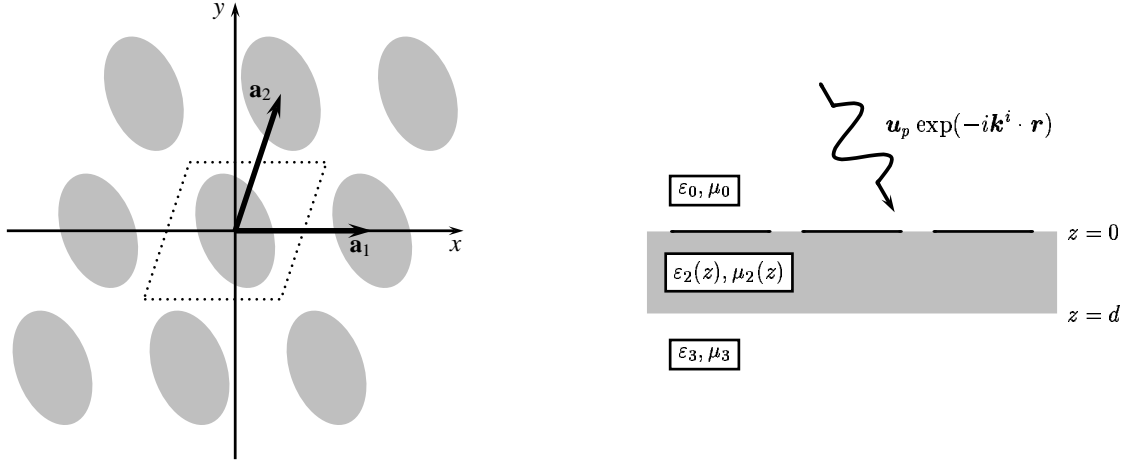


Fig. 1 Left-hand side: metallization, Bravais lattice and elementary cell; right-hand side: cross section.

field components are expressed in components parallel and orthogonal to the direction \mathbf{k}_T/k_T , according to:

$$\hat{\mathbf{E}}_T = -i \frac{\mathbf{k}_T}{k_T} V^e + i(\mathbf{u}_z \times \frac{\mathbf{k}_T}{k_T}) V^h, \quad (3a)$$

$$\hat{\mathbf{H}}_T = -i(\mathbf{u}_z \times \frac{\mathbf{k}_T}{k_T}) I^e - i \frac{\mathbf{k}_T}{k_T} I^h. \quad (3b)$$

In (3), the amplitudes $V^{e,h}(\mathbf{k}_T, z, \omega)$ and $I^{e,h}(\mathbf{k}_T, z, \omega)$ have, apart from a normalizing constant, the proper dimension of a frequency-domain current and voltage along a transmission line. Substituting (3) in the longitudinal part of Maxwell's equations leads to the identification

$$\tilde{\epsilon} \hat{E}_z = -k_T I^e / i\omega, \quad (4a)$$

$$\mu \hat{H}_z = -k_T V^h / i\omega, \quad (4b)$$

where $\tilde{\epsilon}(z, \omega)$ is the usual frequency-domain permittivity.

The expressions in (3) and (4) are substituted in the transverse parts of Maxwell's equations, and again the components parallel and orthogonal to \mathbf{k}_T/k_T are separated. This leads to the desired transmission-line equations. For I^e and V^e , we arrive at

$$\partial_z I^e = i\omega \tilde{\epsilon} V^e - \left(\frac{i\mathbf{k}_T}{k_T} \cdot \hat{\mathbf{J}}_S \right) \delta(z), \quad (5a)$$

$$\partial_z V^e = \frac{-i\gamma^2}{\omega \tilde{\epsilon}} I^e, \quad (5b)$$

where $\gamma = \sqrt{(k_T)^2 - \omega^2 \tilde{\epsilon}(z) \mu(z)}$, with $\text{Re}(\gamma) \geq 0$, and $\text{Im}(\gamma) \leq 0$ when $\text{Re}(\gamma) = 0$. This will be our choice of branch cut in all the square roots appearing in the rest of the paper. For I^h and V^h , the transmission-line equations read

$$\partial_z I^h = \frac{-i\gamma^2}{\omega \mu} V^h + \left(\mathbf{u}_z \times \frac{i\mathbf{k}_T}{k_T} \right) \cdot \hat{\mathbf{J}}_S \delta(z), \quad (6a)$$

$$\partial_z V^h = i\omega \mu I^h. \quad (6b)$$

From (5) and (6), we observe that the Fourier-transformed version of Maxwell's equations indeed has two independent solutions. The first one is determined by I^e and V^e . From (4) it follows that, for this type of solution, only the electric field has a longitudinal component. Therefore, these solutions are indicated as E or TM modes [1]. The second solution is determined by I^h and V^h . For these solutions, only the magnetic field has a longitudinal component. Therefore, they are indicated as H or TE modes.

To arrive at a modular approach for solving the problem, we consider the transmission line equations for $\hat{\mathbf{J}}_S = \mathbf{0}$, and for a unit-amplitude wave incident from $z < 0$. This leads to the situation shown in Figure 2. Since this solution is known as a *Jost solution* [2], we will indicate it by the subscript J . In this manner, we have converted the excitation problem to a plane-wave scattering problem with continuous $I_J^{e,h}(z)$ and $V_J^{e,h}(z)$. In general, the spectral constituents $I_J^{e,h}(z)$ and $V_J^{e,h}(z)$ have to be determined by computational techniques. For a general choice of \mathbf{u}_p the incident field can also be decomposed into an E and a H -mode. The response to both modes is then the corresponding combination of two Jost solutions.

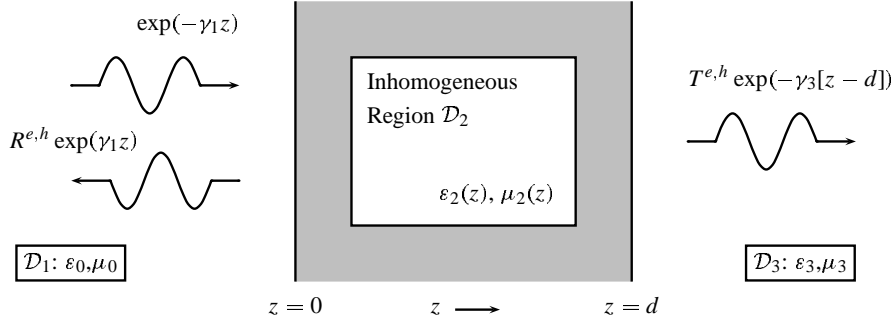


Fig. 2 Jost solution for incidence from $z < 0$.

The E -mode spectral amplitudes corresponding to $\hat{\mathbf{J}}_S$ can now be written in the form

$$I^e(z) = \begin{cases} A \exp(\gamma_1 z) & \text{for } z < 0, \\ B I_J^e(z) & \text{for } z > 0, \end{cases} \quad (7)$$

and

$$V^e(z) = \begin{cases} \frac{\gamma_1 A}{i\omega\epsilon_0} \exp(\gamma_1 z) & \text{for } z < 0, \\ B V_J^e(z) & \text{for } z > 0. \end{cases} \quad (8)$$

Integrating (5a) and (5b) around $z = 0$ then yields the values of the constants A and B . In the integral equation, we need $V^e(0)$, which is given by

$$V^e(0) = \frac{\gamma_1 A}{i\omega\epsilon_0} = \frac{\gamma_1}{i\omega\epsilon_0} \frac{1 - R^e}{2} \left(\frac{i\mathbf{k}_T}{k_T} \cdot \hat{\mathbf{J}}_S \right), \quad (9)$$

where $R^e(k_T, \omega)$ is the plane-wave coefficient for the Jost solution. For H modes, a similar procedure results in

$$V^h(0) = \frac{-i\omega\mu_0}{\gamma_1} \frac{1 + R^h}{2} \left(\mathbf{u}_z \times \frac{i\mathbf{k}_T}{k_T} \right) \cdot \hat{\mathbf{J}}_S. \quad (10)$$

USING PERIODICITY

For an infinite array, we can simplify the analysis by using the fact that the unknown surface current, and any other physical quantity in the configuration, satisfies the Bloch property

$$\mathbf{J}_S(\mathbf{r}_T + n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2) = \exp(-i\mathbf{k}_T^i \cdot [n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2]) \mathbf{J}_S(\mathbf{r}_T) \quad (11)$$

where n_1 and n_2 are arbitrary integers. Functions that satisfy such a condition are a common tool in Solid State Physics, where they are called Bloch functions [3, 4]. From the condition (11), it is obvious that the computation can be restricted to a single elementary cell, e.g., the cell with $n_1 = n_2 = 0$.

The question is how this result can be incorporated in the Fourier representation discussed in the previous section. To achieve this, we use Bloch's theorem, which states that any function that satisfies (11) can be written as the product of an exponential function $\exp(-i\mathbf{k}_T^i \cdot \mathbf{r}_T)$ and a function that has the periodicity of the Bravais lattice. Therefore, let us consider a scalar function $\psi(\mathbf{r}_T)$ with this periodicity. Such a function can be represented by the Fourier series

$$\psi(\mathbf{r}_T) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \Psi_{n_1, n_2} \exp(i[n_1 \mathbf{b}_1 + n_2 \mathbf{b}_2] \cdot \mathbf{r}_T), \quad (12)$$

where \mathbf{b}_1 and \mathbf{b}_2 are reciprocal lattice vectors

$$\mathbf{b}_1 = 2\pi \frac{\mathbf{a}_2 \times \mathbf{u}_z}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{u}_z)}, \quad \mathbf{b}_2 = 2\pi \frac{\mathbf{u}_z \times \mathbf{a}_1}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{u}_z)}. \quad (13)$$

The Fourier coefficients are given by

$$\Psi_{n_1, n_2} = \frac{1}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{u}_z)} \iint_{S_{\text{cell}}} \exp(-i[n_1 \mathbf{b}_1 + n_2 \mathbf{b}_2] \cdot \mathbf{r}_T) \psi(\mathbf{r}_T) dA, \quad (14)$$

where S_{cell} is the volume of an elementary cell in the Bravais lattice. Substituting the results (11) and (14) in the inverse transformation corresponding to (2) then results in

$$\hat{J}_S(\mathbf{k}_T) = \hat{J}_S^0(\mathbf{k}_T) \frac{4\pi^2}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{u}_z)} \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \delta(\mathbf{k}_T - \mathbf{k}_T^i - n_1 \mathbf{b}_1 - n_2 \mathbf{b}_2), \quad (15)$$

where $\hat{J}_S^0(\mathbf{k}_T)$ is a similar Fourier transformation as (2), but only over the lattice cell with $n_1 = n_2 = 0$. Now, we can treat $\hat{J}_S^0(\mathbf{k}_T)$ as the fundamental unknown, and include the remaining ‘‘comb function’’ in the Green’s function.

INTEGRAL EQUATION

Now, we have all the necessary elements to draw up the integral equation. We combine (3a), (9), (10), and (15) to obtain the tangential electric field strength in the plane $z = 0$. We end up with

$$\mathbf{E}_T(\mathbf{r}_T) = \mathbf{E}_T^i(\mathbf{r}_T) + \nabla_T \nabla_T \cdot \iint_{S_0} G_1(\mathbf{r}_T - \mathbf{r}'_T) \mathbf{J}_S(\mathbf{r}'_T) dA' + \iint_{S_0} G_2(\mathbf{r}_T - \mathbf{r}'_T) \mathbf{J}_S(\mathbf{r}'_T) dA' \quad (16)$$

where S_0 is the domain of the metal patch in the cell with $n_1 = n_2 = 0$. $\mathbf{E}_T^i(\mathbf{r}_T)$ is the field that would be present in the layered dielectric in absence of the metallization, and the Green’s functions are given by

$$G_1(\mathbf{r}_T) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \left\{ \frac{\exp(i\mathbf{k}_{n_1, n_2} \cdot \mathbf{r}_T)}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{u}_z)} \left[\frac{1}{k_T^2} \left(\frac{\gamma_1}{i\omega\epsilon_0} \frac{R^e - 1}{2} + \frac{i\omega\mu_0}{\gamma_1} \frac{1 + R^h}{2} \right) \right] \Big|_{\mathbf{k}_T = \mathbf{k}_{n_1, n_2}} \right\} \quad (17)$$

and

$$G_2(\mathbf{r}_T) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \left\{ \frac{\exp(i\mathbf{k}_{n_1, n_2} \cdot \mathbf{r}_T)}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{u}_z)} \left[\frac{i\omega\mu_0}{\gamma_1} \frac{1 + R^h}{2} \right] \Big|_{\mathbf{k}_T = \mathbf{k}_{n_1, n_2}} \right\} \quad (18)$$

with $\mathbf{k}_{n_1, n_2} = \mathbf{k}_T^i + n_1 \mathbf{b}_1 + n_2 \mathbf{b}_2$. Both $G_1(\mathbf{r}_T)$ and $G_2(\mathbf{r}_T)$ are Bloch functions. An advantage of this formulation is that the influence of the layered halfspace is completely described by the reflection coefficients R^e and R^h . When the interest is only in the reflection properties of an FSS, only these coefficients need to be retained from the numerical solution of the transmission line equations (5) and (6). Finally, the integral equation for $\mathbf{J}_S^0(\mathbf{r}_T)$ is obtained by requiring that $\mathbf{E}_T(\mathbf{r}_T) = \mathbf{0}$ for $\mathbf{r}_T \in S_0$.

COMPUTATIONAL APPROACH

We solve the integral equation (16) by the standard method of moments. We write

$$\mathbf{J}_S(\mathbf{r}_T) = \sum_{n=1}^N I_n \mathbf{J}_n(\mathbf{r}_T), \quad (19)$$

where $\mathbf{J}_n(\mathbf{r}_T)$ is a standard expansion function – typically a rooftop [5] or Rao-Wilton-Glisson [6] function – and where the summation runs over the interior edges in the discretization of S_0 . The constant I_n represents the total current across the edge with index n . Next, the integral equation (16) is multiplied by an expansion function $\mathbf{J}_n(\mathbf{r}_T)$, and the product is integrated over its support S_n . Integration by parts then leads to the matrix equation

$$\sum_{n'=1}^N Z_{n, n'} I_{n'} = V_n, \quad (20)$$

for $n = 1, \dots, N$, with

$$Z_{n,n'} = \iint_{S_n} [\nabla_T \cdot \mathbf{J}_n(\mathbf{r}_T)] \iint_{S_{n'}} G_1(\mathbf{r}_T - \mathbf{r}'_T) [\nabla'_T \cdot \mathbf{J}_{n'}(\mathbf{r}'_T)] dA' dA - \iint_{S_n} \mathbf{J}_n(\mathbf{r}_T) \cdot \iint_{S_{n'}} G_2(\mathbf{r}_T - \mathbf{r}'_T) \mathbf{J}_{n'}(\mathbf{r}'_T) dA' dA \quad (21)$$

where ∇'_T is the gradient with respect to \mathbf{r}'_T , and

$$V_n = \iint_{S_n} \mathbf{J}_n(\mathbf{r}_T) \cdot \mathbf{E}^i(\mathbf{r}_T) dA \quad (22)$$

When rooftop functions are used for $\mathbf{J}_n(\mathbf{r}_T)$, the convolution structure of the continuous integral equation (16) is particularly suited for iterative techniques like the conjugate-gradient FFT method [7]. For a general mesh, a direct matrix inversion may be more efficient. In either case, the major part of the computation time is consumed in the determination of the matrix elements $\{Z_{n,n'}\}$. These matrix elements are evaluated in the spectral domain, with the aid of Parseval's theorem. In spite of the weighting procedure, an acceleration of the convergence is needed. We use Poisson's summation and a dedicated extrapolation procedure. The reflection coefficients $R^{e,h}$ are determined by a scattering matrix formalism. Three different algorithms are available to compute the scattering matrix of a continuously stratified layer.

ADDITIONAL METALLIZATION LAYERS

Additional metallic layers can be handled by extending the use of the superposition principle as outlined in the formulation of the problem. The total electric field now consists of $\mathbf{E}^i(\mathbf{r})$ and secondary fields generated by the induced electric currents in the metal layers. Imposing the boundary condition for the tangential electric field at each level then leads to a system of coupled integral equations for the surface currents. Computationally, the extension is that the Jost solution for incidence from $z > d$ must be computed as well. Further, the periodicity of the successive metal layers must be chosen such that it remains possible to identify a common elementary cell.

CONCLUSIONS

With the approach described in this paper, it is possible to model the electromagnetic behavior of a frequency-selective surface of a quite general nature. The modular approach offers considerable freedom in the design of such surfaces. In particular, the Green's function in the generalized integral equation can be constructed entirely from the plane-wave reflection coefficients for the planar dielectric substrate. Also in the computation of the reflection properties, no additional information is required. However, as stated in the introduction, the actual numerical implementation and its verification are still work in progress.

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